R7217

Sub. Code
511301

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

Third Semester

Mathematics

MECHANICS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define degrees of freedom.

2. When do we say a constraint is workless?

3. Obtain the kinetic energy of a particle of mass m which slide without friction on the inside of a small tube which is bent in the form of a circle of radius r and the tube rotates about a vertical diameter with a constant velocity ω.

4. What is the expression for Routhian function?

5. Interpret the problem of finding a stationary value of the function $f(q_1, q_2......q_n)$ subject to n constraints of the form $\phi_j(q_1, q_2.....q_n) = 0$.

6. Write the Hamilton's canonical equations of motion for non holonomic system of the generalized applied force which is not derivable from a potential function.

7. Show that the total derivative of the Hamiltonian with respect to time is $-\dfrac{\partial L}{\partial t}$.

8. Write the Hamilton — Jacobi equation.

9. Quote the Jacobi's identity.

10. Define Poisson bracket.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive Lagrangian form of d'Alembert's principle.

Or

(b) Obtain the expression for the kinetic energy of a system in terms of its motion relative to a fixed point with respect to an arbitrary reference point.

12. (a) Find the differential equations of motion for a spherical pendulum of length l using Lagrangian method

Or

(b) Obtain the Jacobi integral for a translating mass spring system with uniform velocity v_0, unstressed spring length l_o and the elongation x as the generalized coordinate.

13. (a) Show that the Hamiltonian function of a schleronomic system is equal to the total energy.

Or

(b) Using the Hamiltonian procedure, obtain the equations of motion for the Kepler problem in plane polar coordinates.

14. (a) Discuss Hamilton's principal function using the canonical integral.

Or

(b) State and prove Stackel's theorem.

15. (a) Describe the moment transformation.

Or

(b) Apply Lagrange brackets and derive the sufficient condition for a canonical transformation.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Formulate that the rotational kinetic energy of a rotating rigid body is $T_{rot} = \frac{1}{2} I\omega^2$.

17. Define orthogonal system. Show that the orthogonal system is separable.

18. Apply Euler-Lagrange equation to find the curve of brachistochrone problem at minimum time.

19. Apply Hamilton-Jacobi method to solve a mass spring system.

20. Show that the rheonomic transformation $Q = \sqrt{2q}\, e^t \cos p$, $P = \sqrt{2q}\, e^{-t} \sin p$ is canonical. Also find the first type of generating function.

R7218

Sub. Code
511302

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. List any four non-trivial topologies for the set $X = \{a, b, c, d\}$.

2. Define subbasis for a topology.

3. Define standard bounded metric.

4. Define unit ball in $\mathbb{R}^n$.

5. Write the tube lemma.

6. Define isometry of a metric space.

7. Define normal space.

8. Define Lindelöf space.

9. Define locally compact space.

10. Write the countable intersection property.

Part B

$(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If $\mathscr{B}$ is a basis for the topology of X and $\mathscr{C}$ is a basis for the topology of Y, show that the collection.

    $\mathscr{D} = \{B \times C \mid B \in \mathscr{B}$ and $C \in \mathscr{C}\}$ is a basis for the topology of $X \times Y$.

    Or

    (b) Let X be a Hausdorff space; let A be a subset of X. Prove the point x is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

12. (a) State and prove sequence lemma.

    Or

    (b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

13. (a) State and prove maximum and minimum value theorem.

    Or

    (b) Show that compactness implies limit point compactness but not conversely.

14. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable sub collection covering X and there exists a countable subset of X which is dense in X.

    Or

    (b) Prove that a compact Hausdorff space is normal.

15. (a) Let X be a locally compact Hausdorff and let Y be a subspace of X. Prove that if Y is closed in X or open in X, then Y is locally compact.

Or

(b) Let X be a space. Prove that the following are equivalent:

(i) X is completely regular.

(ii) X is homeomorphic to a subspace of a compact Hausdorff space.

(iii) X is homeomorphic to a subspace of a normal space.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation.

$f(a) = (f_\alpha(a))_{\alpha \in J}$,

where $f_\alpha : A \to X_\alpha$ for each α. Let ΠX_α have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous. Also discuss what happens if the box topology is used instead of the product topology.

17. Prove that the cartesian product of connected space is connected.

18. State and prove the Lebesgue number lemma.

19. State and prove the Urysohn lemma.

20. Prove that arbitrary product of compact spaces is compact in the product topology.

R7219

Sub. Code
511303

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

Third Semester

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define critical and non-critical activities.

2. Define minimal spanning tree.

3. Define extreme point of a convex set.

4. Show that the following set is convex:

   $C = \{(x_1, x_2) \mid x_1 \leq 2, x_2 \leq 3, x_1 \geq 0, x_2 \geq 0\}$

5. Define two-person zero sum game.

6. Define saddle point of a game.

7. Write the sufficient condition for a stationary point X_0 to be an extremum.

8. Define control matrix of the constrained derivative method.

9. Define separable function.

10. Define a quadratic programming model.

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11\. (a) Using Dijkstra's algorithm, find the shortest route between 1 and 5 of the following figure 1:

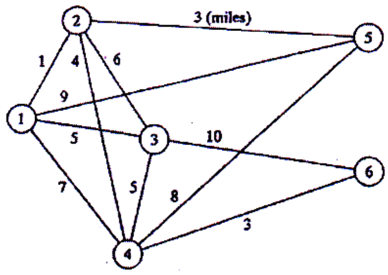

Figure 1:

Or

(b) Write the Floyd's algorithm.

12\. (a) Consider the following linear programming problem:

$\max z = x_1 + 4x_2 + 7x_3 + 5x_4$

Subject to

$$2x_1 + x_2 + 2x_3 + 4x_4 = 10$$
$$3x_1 - x_2 - 2x_3 + 6x_4 = 5$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Generate the simplex tableau associated with the basis $B = (P_1, P_2)$.

Or

(b) Write the revised simplex algorithm.

13\. (a) The following game give A's payoff. Determine the values of p and q that will make the entry (2,2) of the game a saddle point:

	B_1	B_2	B_3
A_1	2	4	5
A_2	10	7	q
A_3	4	p	6

Or

(b) Consider the following game: The pay off is for player A.

	B_1	B_2	B_3
A_1	3	–1	–3
A_2	–2	4	–1
A_3	–5	–6	2

Write the player B's linear program.

14. (a) Find the extremum value of the following function

$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$

Or

(b) Write the KKT condition for the following problem:

$\max f(X) = x_1^2 - x_2^2 + x_1x_3^2$

Subject to

$x_1 + x_2^2 + x_3 = 5,$

$5x_1^2 - x_2^2 - x_3 \geq 2,$

$x_1, x_2, x_3 \geq 0$

15. (a) Write the algorithm for Dichotomous method.

Or

(b) Write the algorithm for Golden section method.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Using maximal flow algorithm, determine the maximal flow in the network (see figure (2))

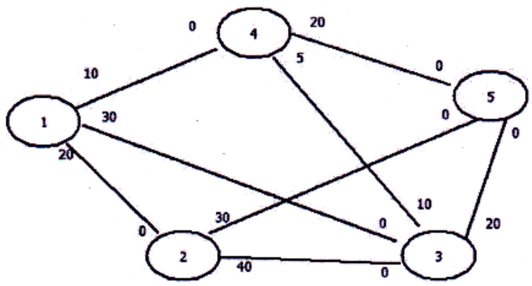

Figure 2:

17. Solve the following linear programming model

$\max = 3x_1 + 5x_2 + 2x_3$

Subject to

$x_1 + x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 43$

$0 \leq x_2 \leq 4,\ 7 \leq x_2 \leq 10,\ 0 \leq x_3 \leq 3.$

18. Solve the following game graphically. The payoff is for player A.

	B_1	B_2
A_1	5	8
A_2	6	5
A_3	5	7

19. Solve the linear programming problem using Jacobian method:

$\max\ x = 2x_1 + 3x_2$

Subject to

$x_1 + x_2 + x_3 = 5$

$x_1 - x_2 + x_4 = 3$

$x_1, x_2, x_3, x_4 \geq 0.$

20. Solve the problem:

$\max z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to

$x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0.$

R7220

Sub. Code
511503

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2022

Third Semester

Mathematics

MULTIVARIATE CALCULUS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define directional derivative of f *at* x.

2. State inverse function theorem.

3. Define null space and projection.

4. State rank theorem.

5. Define primitive mapping.

6. Let D be the 3-cell defined by $0 \le r \le 1,\ 0 \le \theta \le \pi,\ 0 \le \varphi \le 2\pi$. Define $\Phi(r, \theta, \varphi) = (x, y, z)$, where $x = r\sin\theta\cos\varphi,\ y = r\sin\theta\sin\varphi,\ z = r\cos\theta$. Then find the volume of $\Phi(D)$.

7. Define affine k-chain.

8. For $0 \le u \le \pi, 0 \le v \le 2\pi$, define $\sum(u,v) = (\sin u \cos v, \sin u \sin v, \cos u)$. Then show that $\partial \sum = 0$.

9. When a k-form is said to be exact?

10. State Green's theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable in E and there is a real number M such that $\|f'(x)\| \le M$ for every $x \in E$. Then prove that $|f(b) - f(a)| \le M|b - a|$ for all $a \in E, b \in E$.

Or

(b) Suppose f maps an open set $E \subset R^n$ into R^m, f is differentiable at a point $x \in E$. Then prove that the partial derivatives $(D_j f_i)(x)$ exist and $f'(x) e_j = \sum_{i=1}^{m} (D_j f_i)(x) u_i \ (1 \le j \le n)$.

12. (a) Prove that

(i) If I is the identity operator on R^n, then $\det[I] = \det(e_1, \ldots, e_n) = 1$

(ii) det is a linear function of each of the column vectors x_j, if the others are held fixed

(iii) If $[A]_1$ is obtained from $[A]$ by interchanging two columns, then $\det[A]_1 = -\det[A]$

(iv) If $[A]$ has two equal columns, then $\det[A] = 0$.

Or

(b) Suppose f is defined in an open set $E \subset R^2$, suppose that $D_1 f, D_{21}f$ and $D_2 f$ exist at every point of E and $D_{21}f$ is continuous at some point $(a, b) \in E$. Then prove that $D_{21}f$ exists at (a, b) and $(D_{21}f)(a, b) = (D_{21}f)(a, b)$.

13. (a) Discuss the properties of basic k-forms.

Or

(b) Suppose T is a 1 – 1 C' mapping of an open set $E \subset R^k$ into R^k such that $J_T(x) \neq 0$ for all $x \in E$. If f is a continuous function on R^k whose support is compact and lies in $T(E)$, then prove that $\int\limits_{R^k} f(y)\,dy = \int\limits_{R^k} f(T(x))\,|J_T(x)|\,dx$.

14. (a) Suppose E is an open set in R^n, T is a C' mapping of E into an open set $V \subset R^m$. Let ω and λ be k-and m-forms in V, respectively. Then prove that

(i) $(\omega + \lambda)_T = \omega_T + \lambda_T$ if $k = m$

(ii) $(\omega + \lambda)_T = \omega_T \wedge \lambda_T$

(iii) $d\ (\omega_T) = (d\omega)_T$ if ω is of class C' and T is of class C''.

Or

(b) Suppose ω is a k-form in an open set $E \subset R^n$, Φ is a k-surface in E with parameter domain $D \subset R^k$ and Δ is the k-surface in R^k with parameter domain D defined by $\Delta(u) = u (u \in D)$. Then prove that $\int\limits_{\Phi} \omega = \int\limits_{\Delta} \omega_{\Phi}$.

15. (a) Suppose E is an open set in R^3, $u \in C''(E)$ and G is a vector field in E of class C'', the prove that

(i) If $F = \nabla u$ then $\nabla \times F = 0$

(ii) If $F = \nabla \times G$ then $\nabla \cdot F = 0$.

Or

(b) State and prove divergence theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Suppose f maps an open set $E \subset R^n$ into R^m. Then prove that $f \in C'(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \le i \le m, 1 \le j \le n$.

17. State and prove implicit function theorem.

18. Suppose K is a compact subset of R^n and $\{V_\alpha\}$ is an open cover of K. Then prove that there exist functions $\psi_1, ..., \psi_s \in C(R^n)$ such that

(a) $0 \le \psi_i \le 1$ for $1 \le i \le s$

(b) Each ψ_i has its support in some V_α and

(c) $\psi_1(x) + ... + \psi_s(x) = 1$ for every $x \in K$.

19. Explain about the simplexes and chains.

20. If $E \subset R^n$ is convex and open, if $k \ge 1$, if ω is a k-form of class C' in E and if $d\omega = 0$, then prove that there is a $(k-1)$ form λ in E such that $\omega = d\lambda$.